Brass Tacks

An in-depth look at a radio-related topic

Complex power

By now, you're probably acquainted with Ohm's Law, one of the most foundational of all electrical principles (using **V** instead of **E** for the voltage this time):

$$V = I \times R$$

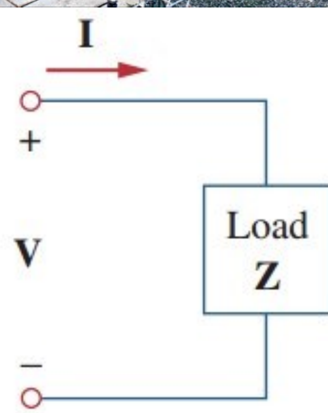
Corollary to Ohm's Law is the calculation for power to the same load:

$$P = I \times V = I^2R$$

Results are typically simple for resistive loads. You might recall that impedance is defined by the complex number

$$Z = R + jX$$

in which **R** is the load resistance, **X** is the load reactance, due to its inductance and capacitance, and **j** is the imaginary unit. If we apply a voltage across a complex impedance, we have instead



$$V = I \times Z$$

$$V = I \times (R + jX)$$

$$V = I \times R + j(I \times X) = IR + jIX$$

But because this **V** is now a complex number, due to the complex impedance **Z**, let's use **S** to represent the complex result of the power calculation

$$S = I \times V = I \times (IR + jIX) = I^2R + jI^2X$$

By convention, the letter **P** is used to represent the resistive power **I^2R**, and **Q** is used to represent the imaginary value **I^2X**. So, the equation becomes

$$S = P + jQ$$

in which

S =	***complex power***	units of **VA**	(volt-amperes)
P =	***real power***	units of **W**	(watts) [aka ***active power*** and ***true power***]
Q =	***reactive power***	units of **VAR**	(volt-amperes reactive)

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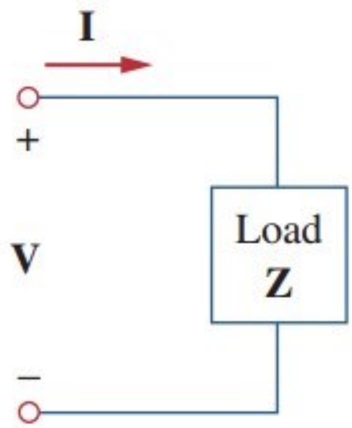
Power triangle and power factor



You might recall that the magnitude of a complex number is found by the square root of the sum of the real portion squared and the imaginary portion squared:

$$Z = R + jX$$

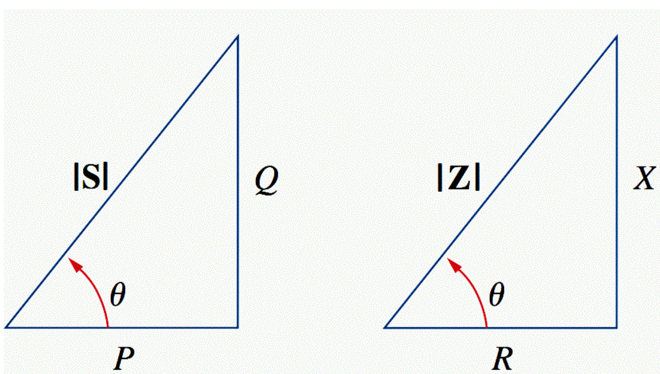
$$|Z| = \sqrt{(R^2 + X^2)}$$

The corresponding power values are derived from the voltage **V** being applied across the load impedance, from the previous page:

$$S = P + jQ$$

$$|S| = \sqrt{(P^2 + Q^2)}$$

In this case, |S| is called the ***apparent power***, and is still measured in **VA**. And because of this Pythagorean relationship, we can represent these values in a right triangle:



From this ***power triangle***, another value that's relevant to the discussion of complex power is the ***power factor***, labeled ***PF***, which is the ratio of the real power **P** to the apparent power **|S|**, providing an idea of how much of the generated (apparent) power is actually being put to useful work (real power). The ***power factor angle***, labeled ***θ*** (also known as the ***phase angle***), is the same angle as the load impedance angle, between the real portion and the magnitude. Because of this being a right-triangle, the power factor is the cosine of the power factor angle:

$$PF = \cos(\theta) = R\ /\ |Z| = P\ /\ |S|$$

and therefore

$$P = |S| \times \cos(\theta)$$

In essence, the power factor is the cosine of the phase angle between the voltage sinusoid and the current sinusoid. From this power factor ratio, we could see that the higher the ratio (and the less the angle), the more generated power is being put to useful work. If the load is inductive, **X** (and therefore **Q**) will be positive; if it's capacitive, **X** (and therefore **Q**) will be negative. **Q** is positive or negative, depending on the *net reactance*.

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Reactive power

One of the electrical mysteries that tend to stump or confuse the most people regarding complex power is the concept of ***reactive power***, which is neither dissipated nor consumed, but is still necessary to fulfill the mission of complex power. Because of inductance and capacitance in the load, the power is repeatedly stored and released in these components, but is *wattless and non-productive*.

A bit of a stretch, reactive power can be thought of like water over a mill water wheel. The river supplies the water, which is necessary to provide energy to the wheel, and the water simply returns to the river, but with less energy. The returning water is *waste* to the mill, because it can no longer provide any useful energy, but was still necessary for the process.

The reactive power still needs to be paid for, because the entire power transfer-and-exchange process could not occur without it, so the power company (or your power supply) must still generate it.

What that means to you is that if your business runs a lot of motors, that large inductive load translates into a potentially low power factor. To accomplish the same amount of work as without the motors, your business will need to draw more power from the utility company, costing you more. Today, some businesses invest in *power factor correction*, by installing industrial-sized capacitor banks to offset the reactive power requirement, since most power factor imbalances are due to inductive loads.

In the diagram to the right, the many motors in a business have resulted in a low power factor (due to the large $\mathbf{Q_{L1}}$, the entire blue ray from the solid green ray to the solid red ray) which is

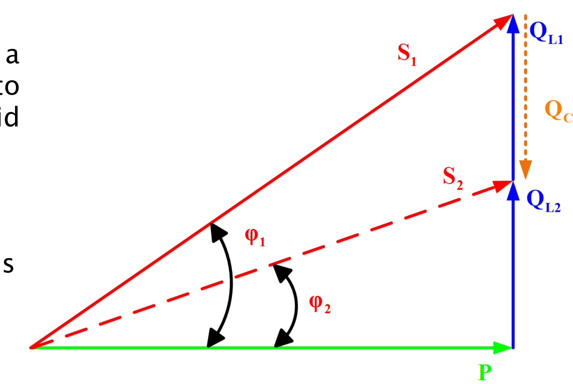
$$\mathbf{P / S_1}$$

But after adding a capacitor bank, which contributes

$$\mathbf{-Q_C}$$

the new (greater) power factor is

$$\mathbf{P / S_2}$$



Through all of this, the value of **P** (the actual amount of the drawn power put to useful work) hasn't changed. The value of **S** is lower because of $\mathbf{-Q_c}$, making it *appear* like the business is using less power, and hence the name *apparent power*. The closer the capacitive and inductive load (reactances) adds to zero, the higher the power factor and less the load needs to draw from the power company, and the lower the power bill.

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Which do you pay for on your utility bill, the apparent power, the real power, the reactive power, or all three? Most utility companies tend to bill *residences* and small businesses for real power and *commercial industries* for apparent power. Some charge each industrial customer for real power, and then add a *power factor penalty* to the bill when their power factor falls below a predetermined threshold.

Average power and real power

A natural result of power discussions is the concept of ***average power***, the total amount of energy consumed or dissipated over a specific period of time, $\mathbf{P_{ave} = \Delta E / \Delta t}$. The quantity is used to give consumers a way to understand how much energy they had demanded during the past month, for example.

While the time period (**Δt**) is often simple to calculate or estimate, the total energy (**ΔE**) might not be so intuitive, and its calculation differs between energy types, such as *thermal* (natural gas), *light* (lamps and solar), *mechanical* (think vehicles, A/C, and refrigerator compressors), *chemical* (batteries and wood-burning stoves), and for our discussion, *electrical*. The precise calculation for average electrical power is

$$P_{ave} = \frac{1}{T}\int_0^T p(t)dt$$

It's an integral because the power consumption changes from one split moment to the next. The function ***p(t)*** is called ***instantaneous power***, and is calculated from

$$\boldsymbol{p(t) = i(t) \times v(t)}$$

similar to the **P = I x V** equation we're already familiar with, except that

$$\boldsymbol{v(t)} = \mathbf{V_0 \sin(2\pi ft)} \quad \text{and} \quad \boldsymbol{i(t)} = \mathbf{I_0 \sin(2\pi ft \pm \text{ø})}$$

Because the current is the result of the voltage divided by the load impedance, the voltage and current will have the same frequency ***f***, but differ by phase angle **ø**. These two functions are known as *instantaneous voltage* (the peak voltage V_0 at any moment in time, modified by the sine function) and *instantaneous current*, (the peak current I_0 at any moment in time, modified by the sine function shifted by an angle of **ø**), respectively. Solving the integration gives the average power equivalent:

$$P_{\text{ave}} = \frac{1}{2} I_0 V_0 \cos\phi$$

Because the average power is related to the peak current and peak voltage by the power factor, the average power is also known as ***real power***, also called ***true power***, ***active power***, and a few others. *It is power that is consumed by a load*.

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From our discussion on RMS, we also know that

$$I_{rms} = \frac{1}{\sqrt{2}} I_0 \quad \text{and} \quad V_{rms} = \frac{1}{\sqrt{2}} V_0$$

Multiply the two together

$$I_{rms} \ \mathbf{x} \ V_{rms} = \frac{1}{\sqrt{2}} I_0 \ \mathbf{x} \ \frac{1}{\sqrt{2}} V_0 = \frac{1}{2} I_0 V_0$$

therefore,

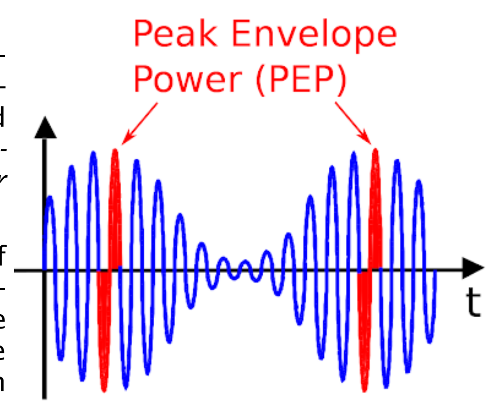
$$P_{ave} = I_{rms} V_{rms} \cos \phi$$

proving the average power (and therefore the real power) can be calculated from the RMS values for current and voltage, if the phase angle is known.

By the way, the quantity *RMS power* can also be calculated, but has no meaningful or practical use in the physical world, and so is omitted here. In some discussions, when the term RMS power is used, it's typically a mis-speak for average power, largely out of misunderstanding of the terms.

Peak envelope power

One application for the use of average power is the calculation of ***peak envelope power*** (PEP), a value immediately relevant to the world of amateur radio. It's mentioned in the FCC rules [Part 97.3(b)(9), Part 97.313(b), etc.], but many struggle with the concept, so understanding it might require a little visualization. It's often easier to illustrate the property by examining an AM (amplitude modulation) radio signal.



The drawing to the right displays the output power signal of an AM radio transmitter, showing the higher-frequency carrier wave that has been modified (modulated) by a lower-frequency sine wave. *Each oscillation of the carrier signal is a result of the average power of the signal in that cycle.*

The outline made from connecting the highest points of the carrier wave together, and another outline from connecting the lowest points, form the *power envelope*. The highest points in the envelope define the peak envelope power of the signal, marked in red. When observed with an oscilloscope, the displayed peak envelope voltage can be used to calculate the PEP of your transmitter by **PEP = ½(PEV)² / 50**

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Brief rectangular / polar / phasor / vector / sinusoid overview

Many discussions, diagrams, and lessons present complex numbers in rectangular format, such as **a+jb**, and/or polar format, such as $\mathbf{c}\angle 30^\circ$. But other terms, such as phasor, vector, and more, seem to get thrown into the mix to describe them, so let's try and sort them all out.

Let the peak voltage be $V_0 = V_m = 120\sqrt{2}$ **V** at **60 Hz** and the phase angle be 16° **:**

Sinusoidal form $v(t) = 120\sqrt{2}\sin(120\pi t + 16^\circ)$ **V** at any time **t**

$$V_{rms} = (V_0) / (\sqrt{2}) = (120\sqrt{2}) / (\sqrt{2}) = 120\ V_{rms}$$

$$V_{real} = 120\cos(16^\circ) = 115.35\ V \text{ and } V_{img} = 120\sin(16^\circ) = 33.1\ V$$

$$V_{rms} = \sqrt{[(V_{real})^2 + (V_{img})^2]} = 120\ V_{rms}$$

Rectangular (also called *vector*) form $V = V_{real} + jV_{img}$ $V = 115.35 + j33.1\ V$

Polar (also called *phasor*) form $V = V_{rms}\angle\theta$ $V = 120\angle 16^\circ\ V_{rms}$

Equation recap

Here is a list of some of the concluding equations that define the different aspects of complex power, including some that were not previously mentioned:

Complex power	$S = P + jQ$	
Apparent power	$\lvert S\rvert = I_{rms}V_{rms}$	**[measured in VA]**
Real power	$P = \lvert S\rvert\cos(\theta)$	**[measured in W]**
Reactive power	$Q = \lvert S\rvert\sin(\theta)$	**[measured in VAR]**
Power factor	$PF = \cos(\theta) = P / \lvert S\rvert$	
Average power	$P = I_{rms}V_{rms}\cos(\theta) = \lvert S\rvert\cos(\theta)$	**[in W]**

Summary

Complex power is the power supplied to a load impedance by an AC voltage. If the load impedance consists of a non-zero reactance, complex power consists of apparent power, real power, and reactive power. The ratio of the real power to the apparent power is called the power factor, and is equal to the cosine of the phase angle between the voltage and resulting current signals. Because reactive power is neither consumed nor dissipated, it can't be used for any useful work, but must still be generated. Real power is both consumed and is useful, and is the same as average power, which is a quantity of energy over a specific time period.

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